Effects of Grain Size on the Bremsstrahlung Spectrum of Electron-Dust Grain Collisions in Dusty Plasmas

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The grain size effects on the bremsstrahlung emission spectrum due to nonrelativisitc electron-dust grain collisions are investigated in dusty plasmas. Using the Born approximation for the initial and final states of the projectile electron, the bremsstranhlung radiation cross section is obtained as a function of the grain size, dust charge, Debye radius, collision energy, and radiation photon energy. It is found that the effects of the grain size enhance the bremsstrahlung radiation cross section, especially for soft-photon radiations. The effect of the Debye radius on the bremsstrahlung cross section is found to be increased with an increase of the magnitude of the charge number of the dust grain. In addition, the grain size effect on the bremsstrahlung spectrum is found to be more significant for highly charged dusty grains.

Key words: Dust Bremsstrahlung; Dusty Plasmas.

Recently, there has been a substantial interest in the dynamics of plasmas containing highly charged aerosol or charged dust grains, including collective effects and strong electrostatic interaction between the charged components [1 – 6]. It has been shown that dust-plasma interactions are ubiquitous in many astrophysical and laboratory dusty plasmas. Various physical processes in dusty plasmas have been extensively investigated in order to get information on plasma parameters in dusty plasmas [2 – 4]. Among several collision and radiation processes in plasmas, the bremsstrahlung process [7 – 9] has received much attention since the continuum bremsstrahlung spectrum due to the binary encounters has been widely used in plasma diagnostics. Furthermore, it is expected that the electron-dust grain bremsstrahlung process would be considerably different from the electron-ion bremsstrahlung process due to the enormous size of a dust grain. However, to the best of our knowledge, the finite grain size effect on the electron-dust grain bremsstrahlung process in dusty plasmas has not been investigated yet. In addition, it could be anticipated that the information on the size of dust grains would be deduced from the bremssstrahlung emission spectrum in dusty plasmas. Thus, in the present paper we investigate the effects of the grain size on the bremsstrahlung emission spectrum due to the electron-dust grain interactions in dusty plasmas. The nonrelativistic Born analysis [10] is employed for both the initial and final states of the projectile electron in order to derive the electron-dust grain bremsstrahlung radiation cross section in dusty plasmas as a function of the grain size, dust charge, Debye radius, collision energy, and radiation photon energy.

Using the second-order nonrelativistic perturbation analysis [9], the differential electron bremsstrahlung cross section $\mathrm{d}\sigma_\mathrm{b}$ for the interaction potential $V(\mathbf{r})$ in the solid angle $\mathrm{d}\Omega$ can be written as

$$\frac{\mathrm{d}\sigma_\mathrm{b}}{\mathrm{d}\Omega} = \mathrm{d}\sigma_\mathrm{C}\frac{\mathrm{d}W_\omega}{\mathrm{d}\Omega}, \tag{1}$$

where the differential elastic scattering cross section $\mathrm{d}\sigma_\mathrm{C}$ is represented by

$$\mathrm{d}\sigma_\mathrm{C} = \frac{1}{2\pi\hbar v_0^2}|\tilde{V}(\mathbf{K})|^2 K\mathrm{d}K. \tag{2}$$

Here $\hbar$ is the rationalized Planck constant, v_0 is the initial velocity of the projectile electron, $\tilde{V}(\mathbf{K})$ is the Fourier transformation of the interaction potential

$$\tilde{V}(\mathbf{K}) = \int \mathrm{d}^3\mathbf{r}\mathrm{e}^{-\mathrm{i}\mathbf{K}\cdot\mathbf{r}}V(\mathbf{r}), \tag{3}$$

$\mathbf{K}$ $(= \mathbf{k}_0 - \mathbf{k}_f)$ is the momentum transfer, and $\mathbf{k}_0$ and $\mathbf{k}_f$ are, respectively, the wave vectors of the initial and final states of the projectile electron, and $\mathrm{d}W_\omega/\mathrm{d}\Omega$ represents the differential probability of emitting a photon of a frequency between ω and $\omega+\mathrm{d}\omega$ in the solid angle:

$$\frac{\mathrm{d}W_\omega}{\mathrm{d}\Omega}=\frac{\alpha}{4\pi^2}\left(\frac{\hbar}{mc}\right)^2\sum_{\hat{\boldsymbol{e}}}|\hat{\boldsymbol{e}}\cdot\mathbf{K}|^2\frac{\mathrm{d}\omega}{\omega}. \tag{4}$$

Here, α $(=e^2/\hbar c\cong 1/137)$ is the fine structure constant, m the electron mass, c the velocity of the light, and $\hat{\boldsymbol{e}}$ the unit photon polarization vector. Since the summation over photon polarizations provides the angular distribution factor $\sin^2\Theta$ in the $|\hat{\boldsymbol{e}}\cdot\mathbf{K}|^2$ term in (4), where Θ is the angle between the momentum transfer $\mathbf{K}$ and initial wave vector $\mathbf{k}_0$, the bremsstrahlung cross section, obtained by integrating over all directions of the radiation photon, is then

$$\mathrm{d}\sigma_\mathrm{b}=\frac{\alpha}{3\pi^2(mc^2)^2}\frac{1}{\beta_0^2}|\tilde{V}(\mathbf{K})|^2K^3\mathrm{d}K\frac{\mathrm{d}\omega}{\omega} \tag{5}$$

with $\beta_0=v_0/c$. In spherical polar coordinates with their origin at the centre of the dust grain, the interaction potential $V_{\mathrm{e-d}}(r)$ between the electron and stationary dust grain with the charge Q_d $(=Ze)$ in dusty plasmas is customary represented by the Yukawa-type Debye-Hückel form [2]

$$V_{\mathrm{e-d}}(r)=-\frac{Ze^2}{r}\mathrm{e}^{-r/\lambda_\mathrm{D}}, \tag{6}$$

where Z is the charge number of the dust grain and λ_D is the Debye radius of the background dusty plasma. Recently, there has been significant interest in dusty plasmas encompassing elongated charged dust grains [6, 11]. However, for the sake of simplicity, the dust grains are assumed to have spherical shapes in this work. For typical circumstances of astrophysical and laboratory dusty plasmas, it has been shown that $Z\approx -1000$, $\lambda_\mathrm{D}/a\approx 5-100$, and $a\approx 0.01-1$ μm [1], where a is the radius of the dust grain. After some mathematical manipulations, the Fourier transformation of the electron-dust grain interaction potential is obtained as

$$\tilde{V}(K)=\int_{r\geq a}\mathrm{d}^3\mathbf{r}V_{\mathrm{e-d}}(r)\mathrm{e}^{-\mathrm{i}\mathbf{K}\cdot\mathbf{r}}= -\frac{4\pi Ze^2a^2}{\bar{K}}\left[\mathrm{e}^{-1/\bar{\lambda}_\mathrm{D}}\frac{(1/\bar{\lambda}_\mathrm{D})\sin\bar{K}+\bar{K}\cos\bar{K}}{1/\bar{\lambda}_\mathrm{D}^2+\bar{K}^2}\right], \tag{7}$$

where the lower-cutoff in the radial integration has been introduced due to the finite size of the dust grain; $\bar{K}$ $(\equiv Ka)$ is the scaled momentum transfer, and $\bar{\lambda}_\mathrm{D}$ $(\equiv\lambda_\mathrm{D}/a)$ is the scaled Debye radius. The bremsstrahlung cross section is then given by

$$\mathrm{d}\sigma_\mathrm{b}=\frac{16}{3}\frac{\alpha^3Z^2a_0^2}{\bar{E}}\left[\mathrm{e}^{-1/\bar{\lambda}_\mathrm{D}}\frac{(1/\bar{\lambda}_\mathrm{D})\sin\bar{q}+\bar{q}\cos\bar{q}}{1/\bar{\lambda}_\mathrm{D}^2+\bar{K}^2}\right]^2 \cdot\bar{K}\mathrm{d}\bar{K}\frac{\mathrm{d}\omega}{\omega}, \tag{8}$$

where $\bar{E}$ $(\equiv mv_0^2/2Ry)$ is the scaled energy of the projectile electron, m is the mass of the electron, Ry $(=me^4/2\hbar^2\approx 13.6$ eV$)$ is the Rydberg constant, and a_0 $(=\hbar^2/me^2)$ is the Bohr radius of the hydrogen atom. It has been known that the bremsstrahlung emission spectrum would be explored through the bremsstrahlung radiation cross section [12] $\mathrm{d}^2\chi_\mathrm{b}/\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{K}\equiv(\mathrm{d}\sigma_\mathrm{b}/\hbar\mathrm{d}\omega\mathrm{d}\bar{K})\hbar\omega$, where $\bar{\varepsilon}$ $(\equiv\varepsilon/Ry)$ is the scaled radiation photon energy and ε $(=\hbar\omega)$ is the photon energy. After some mathematical manipulations, the bremsstrahlung radiation cross section $\mathrm{d}^2\chi_\mathrm{b}/\mathrm{d}\bar{\varepsilon}$ in units of πa_0^2 due to the electron-dust grain interaction in dusty plasmas is obtained in the following form:

$$\begin{aligned}&\frac{\mathrm{d}^2\chi_\mathrm{b}(Z,\bar{E},\bar{\varepsilon},\bar{\lambda}_\mathrm{D},a)}{\mathrm{d}\bar{\varepsilon}}/\pi a_0^2=\frac{16}{3\pi}\frac{\alpha^3Z^2}{\bar{E}}\int_{\bar{K}_\mathrm{min}}^{\bar{K}_\mathrm{max}}\mathrm{d}\bar{K}\bar{K}\\&\cdot\left[\mathrm{e}^{-1/\bar{\lambda}_\mathrm{D}}\frac{(1/\bar{\lambda}_\mathrm{D})\sin\bar{K}+\bar{K}\cos\bar{K}}{1/\bar{\lambda}_\mathrm{D}^2+\bar{K}^2}\right]^2=\frac{4}{3\pi}\frac{\alpha^3Z^2}{\bar{E}}\mathrm{e}^{-2/\bar{\lambda}_\mathrm{D}}\\&\cdot\left\{\frac{2\cos(2\bar{K}_\mathrm{max})}{1+(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{max})^2}-\frac{2\cos(2\bar{K}_\mathrm{min})}{1+(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{min})^2}\right.\\&-\frac{2(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{max})\sin(2\bar{K}_\mathrm{max})}{1+(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{max})^2}+\frac{2(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{min})\sin(2\bar{K}_\mathrm{min})}{1+(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{min})^2}\\&+\ln\left[\frac{1+(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{max})^2}{1+(\bar{\lambda}_\mathrm{D}\bar{K}_\mathrm{min})^2}\right]+\left(1+\frac{2}{\bar{\lambda}_\mathrm{D}}\right)\left[\cosh\left(\frac{2}{\bar{\lambda}_\mathrm{D}}\right)\right.\\&\left.+\sinh\left(\frac{2}{\bar{\lambda}_\mathrm{D}}\right)\right]\left[Ci\left(-\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}+2\bar{K}_\mathrm{max}\right)-Ci\left(-\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}\right.\right.\\&\left.+2\bar{K}_\mathrm{min}\right)+Ci\left(\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}+2\bar{K}_\mathrm{max}\right)-Ci\left(\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}+2\bar{K}_\mathrm{min}\right)\\&+\mathrm{i}\left(Si\left(\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}-2\bar{K}_\mathrm{max}\right)-Si\left(\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}-2\bar{K}_\mathrm{min}\right)\right.\\&\left.\left.\left.+Si\left(\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}+2\bar{K}_\mathrm{max}\right)-Si\left(\frac{2\mathrm{i}}{\bar{\lambda}_\mathrm{D}}+2\bar{K}_\mathrm{min}\right)\right)\right]\right\},\end{aligned} \tag{9}$$

where $\bar{K}_\mathrm{min}[\equiv(k_0-k_\mathrm{f})a]=(\bar{a}/\sqrt{2})(\sqrt{\bar{E}}-\sqrt{\bar{E}-\bar{\varepsilon}})$ is the scaled minimum momentum transfer, $\bar{K}_\mathrm{max}[\equiv(k_0+$

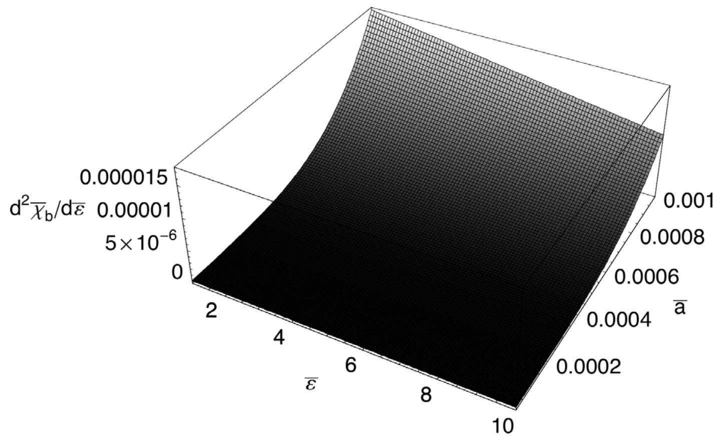

Fig. 1. The three-dimensional plot of the scaled bremsstrahlung radiation cross section $d^2\bar{\chi}_b/d\bar{\varepsilon}$ in units of πa_0^2 as a function of the scaled photon energy $\bar{\varepsilon}$ and the scaled radius of the dust grain $\bar{a}$ for $\bar{E}=20$, $\bar{\lambda}_D=50$, and $Z=-500$.

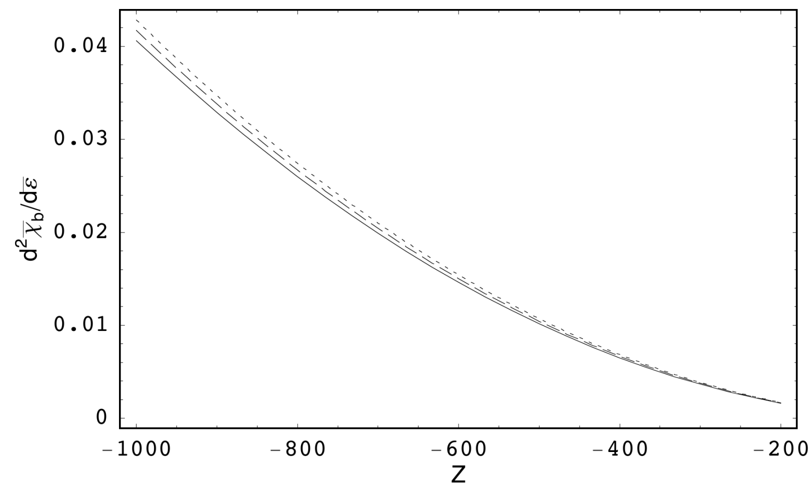

Fig. 2. The scaled bremsstrahlung radiation cross section $d^2\bar{\chi}_b/d\bar{\varepsilon}$ as a function of the charge number Z of the dust grain for $\bar{E}=20$, $\bar{\varepsilon}=5$, and $a=0.1$ μm; solid line, $\bar{\lambda}_D=30$; dashed line, $\bar{\lambda}_D=50$; dotted line $\bar{\lambda}_D=150$.

$k_f)a] = (\bar{a}/\sqrt{2})(\sqrt{\bar{E}}+\sqrt{\bar{E}-\bar{\varepsilon}})$ is the scaled maximum momentum transfer, $\bar{a}$ ($\equiv a/a_0$) is the scaled radius of the dust grain, and the special functions $Ci(z)$ ($=-\int_z^\infty dt \cos t/t$) and $Si(z)$ ($=\int_0^z dt \sin t/t$) are, respectively, the cosine and sine integrals [13].

In order to investigate the grain size effects on the bremsstrahlung emission spectrum due to nonrelativisitc electron-dust grain interactions in dusty plasmas, we set $\bar{E}\gg 1$ since the Born approximation is known to be reliable for $v_0 \gg ac$ [9]. Figure 1 shows the three-dimensional plot of the scaled bremsstrahlung radiation cross section $d^2\bar{\chi}_b/d\bar{\varepsilon}$ [$\equiv (d^2\chi_b/d\bar{\varepsilon})/\pi a_0^2$] in units of πa_0^2 for the interaction of an electron with a negatively charged dust grain in dusty plasmas as a function of the scaled photon energy $\bar{\varepsilon}$ and scaled radius of the dust grain $\bar{a}$. From this figure, it is shown that the effects of the grain size significantly enhance the bremsstrahlung radiation cross section, especially for the case of soft-photon radiations. Thus, the bremsstrahlung radiation cross section is expected to be greater for a bigger size of the dust grain even if the dust charge has been fixed. Figure 2 presents the scaled bremsstrahlung radiation cross section $d^2\bar{\chi}_b/d\bar{\varepsilon}$ as a function of the charge number Z of dust grain for various values of the Debye radius. It is found that the bremsstrahlung radiation cross section decreases with a decrease of the magnitude of the charge number of the dust grain. It is also found that the bremsstrahlung cross section increases with an increase of the Debye radius. In addition, the effect of the Debye radius on the bremsstrahlung cross section is found to be increased with increasing the magnitude of the charge number of the dust grain. Figure 3 presents the three-dimensional plot of the scaled bremsstrahlung radiation cross section $d^2\bar{\chi}_b/d\bar{\varepsilon}$ as a function of the charge number Z of a dust grain and the scaled radius of the dust grain $\bar{a}$.

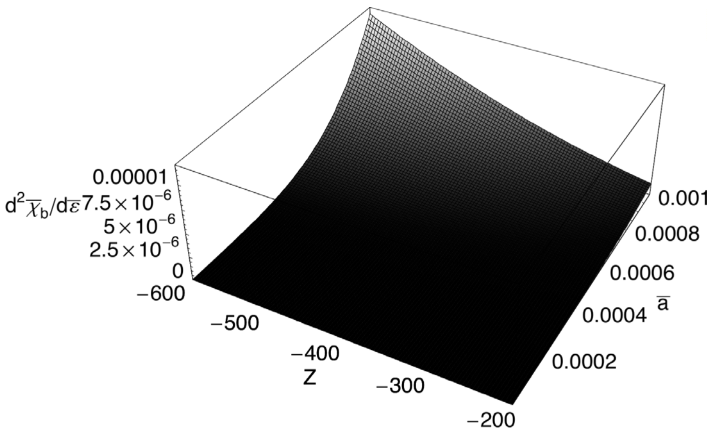

Fig. 3. The three-dimensional plot of the scaled bremsstrahlung radiation cross section $\mathrm{d}^2\bar{\chi}_\mathrm{b}/\mathrm{d}\bar{\varepsilon}$ as a function of the charge number Z of the dust grain and the scaled radius of the dust grain $\bar{a}$ for $\bar{E} = 10$, $\bar{\varepsilon} = 2$, and $\bar{\lambda}_\mathrm{D} = 50$.

As shown, it is also found that the grain size effect on the bremsstrahlung spectrum is more significant for the case of highly charged dusty grains. Hence, we have found that the grain size effect plays a significant role in the electron-dust grain bremsstrahlung process in dusty plasmas containing highly charged dusty grains. These results would provide useful information on the bremsstrahlung emission spectrum due to the electron-dust grain interactions and also show the possibility to ascertain the size of the dust grain in dusty plasmas using plasma spectroscopy.

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